AAS-003-001408 Seat No. ________

B. Sc. (Sem. IV) (CBCS) Examination

April / May – 2016

Mathematics : Paper-BSMT-401(A)

(Adv. Calculus & Linear Algebra)

Faculty Code : 003

Subject Code : 001408

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.

(ii) Write answers of M.C.Q. in main answer sheet.

1 Attempt all M.C.Q.s : **20**

(1) If $f(x,y) = x \ \tan\frac{y}{x}$ then $f_x(4,\pi) =$ ______.

(A) $1-\frac{\pi}{2}$ (B) 2

(C) $\frac{\pi}{2}-1$ (D) $1-\frac{\pi}{4}$

(2) If $u = \sin\left(\frac{x^2+y^2}{xy}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ______.

(A) 0 (B) 1

(C) 2 (D) –1

(3) Total differential dz at (x, y) of $Z = x^2 + xy + y^2$ is

(A) $(x+2y)\delta_x + (2x+y)\delta_y$ (B) $(2x+y)\delta_x + (x+2y)\delta_y$

(C) $(x+y)\delta_x + (2x+y)\delta_y$ (D) $(2x+y)\delta_x + (x+y)\delta_y$

(4) If $u = \sqrt{x^2 - xy}$ then $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \cdot \partial y} + y^2 \cdot \frac{\partial^2 u}{\partial y^2} =$ ________

(A) 0 (B) 1

(C) 2 (D) –1

(5) If $f(x, y) = x^2 + 2y^2 - x$ then critical point of f is

(A) $\left(\frac{1}{2}, 0\right)$ (B) $\left(0, \frac{1}{2}\right)$

(C) $(1, 4)$ (D) $(4, 1)$

(6) If $x = r\cos\theta,\ y = r\sin\theta$ then $\frac{\partial(x, y)}{\partial(r, \theta)} =$ _____.

(A) 0 (B) r

(C) θ (D) 1

(7) If $x = p\cos\theta,\ y = p\sin\theta, z = z$ then $\frac{\partial(x, y, z)}{\partial(p, \theta, z)} =$ ______.

(A) p (B) $\frac{1}{p}$

(C) $p\sin\theta$ (D) $p\cos\theta$

(8) In usual notation, $grad\,\phi =$ _____.

(A) $\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}$

(B) $\left(\frac{\partial\phi}{\partial x},\frac{\partial\phi}{\partial y},\frac{\partial\phi}{\partial z}\right)$

(C) $f_1\hat{i}+f_2\hat{j}+f_3\hat{k}$

(D) $\nabla\times\bar{f}$

(9) $\phi(x,y,z)=x^2y+y^2z+z^2$ then unit normal at point (1,1,1) is

(A) $\frac{1}{\sqrt{22}}\left(3\hat{i}+3\hat{j}+\hat{k}\right)$

(B) $\frac{1}{\sqrt{22}}\left(3\hat{i}+2\hat{j}+2\hat{k}\right)$

(C) $\frac{1}{\sqrt{22}}\left(3\hat{i}+3\hat{j}+2\hat{k}\right)$

(D) $\frac{1}{\sqrt{22}}\left(3\hat{i}+\hat{j}+2\hat{k}\right)$

(10) If a vector function

$\bar{F}=\left(x^2z-axyz\right)\hat{i}+\left(xy-3x^2yz\right)\hat{j}+\left(yz^2-xz\right)\hat{k}$ is solenoidal then

$a =$ ________.

(A) 2

(B) –2

(C) 0

(D) 1

(11) $\iint_R xy\,dy\,dx =$ ________, where

$R=\left\{(x,y)\,|\,x^2\le y\le\sqrt{x},\ 0\le x\le 1\right\}$.

(A) 12

(B) $\frac{1}{12}$

(C) $\frac{1}{14}$

(D) 14

(12) Value of $\int_0^a \int_0^b \int_0^c (x+y+z)\,dx\,dy\,dz$ is

(A) $\frac{abc}{2}(a+b)$ (B) $\frac{abc}{2}(b+c)$

(C) $\frac{abc}{2}(a+b+c)$ (D) $\frac{abc}{2}(a+c)$

(13) $\int_0^1 \int_0^1 \frac{dy\,dx}{\left(1+x^2\right)\left(1+y^2\right)} =$ ________.

(A) $\frac{\pi}{16}$ (B) $\frac{\pi^2}{8}$

(C) $\frac{\pi^2}{16}$ (D) $\frac{\pi^2}{4}$

(14) $\int_c \frac{dx}{x+y} =$ ________, where $C : x = at^2, y = 2at, 0 \le t \le 2$.

(A) log2 (B) log8

(C) log4 (D) log16

(15) The general form of a line integral is

(A) $\iint_c P(x,y)\,dx + Q(x,y)\,dy$ (B) $\int_c P\,dx - Q\,dy$

(C) $\int_c Q\,dx - P\,dy$ (D) $\int_c P(x,y)\,dx + Q(x,y)\,dy$

(16) $\int_{(1,1)}^{(2,3)} x\,dy =$ __________.

(A) -3 (B) 2

(C) 3 (D) -2

(17) In the surface integral $\iint_{s} \overline{V} \cdot \overline{n}\, d\sigma,\ \overline{n} =$ _____.

(A) $\dfrac{div\,\phi}{|div\,\phi|}$ (B) $\dfrac{curl\,\phi}{|curl\,\phi|}$

(C) $\dfrac{grad\,\phi}{|grad\,\phi|}$ (D) Unit Vector

(18) In usual notations $\left\lceil \dfrac{1}{4} \right. \left\lceil \dfrac{3}{4} \right. =$ ______.

(A) 2π (B) $\sqrt{2}\,\pi^2$

(C) $2\pi^2$ (D) $\sqrt{2}\,\pi$

(19) If $u = (2,1,-1) \in R^3$, where R^3 is an Euclidean space then $\|u\| =$ ______.

(A) $\sqrt{5}$ (B) 5

(C) 6 (D) $\sqrt{6}$

(20) V is an inner product space and $\overline{u}, \overline{v} \in V$ and $\alpha \in R$ then $\|\alpha u\| =$ ________.

(A) $\alpha\,\|u\|$ (B) $\|\alpha\|\,\|u\|$

(C) $|\alpha|\,|u|$ (D) $|\alpha|\,\|u\|$

2 (a) Attempt any three : **6**

(1) If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ then find $\frac{\partial^2 u}{\partial x \partial y}$.

(2) If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$ the find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

(3) If $f(x, y, z) = \sqrt{x^2+y^2+z^2}$ then find Approximate value of $f(1.9, 2.01, 4.8)$.

(4) For spherical co-ordinate $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ then find $J\left(\frac{x, y, z}{r, \theta, \phi}\right)$.

(5) In usual notation prove that $div(\phi\overline{f}) = \phi\, div\,\overline{f} + \overline{f}\cdot grad\,\phi$. where $\overline{f}$ is vector function and ϕ is scalar function.

(6) For a scalar function ϕ on D, Prove that $curl(grad\,\phi) = \overline{0}$.

(b) Attempt any three : **9**

(1) If $u = e^{xyz}$ then find $\frac{\partial^3 u}{\partial x \partial y \partial z}$.

(2) If z is a homogeneous function of x and y of degree n and $z = f(u)$ then prove that $x\cdot\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n\frac{f(u)}{f'(u)}$.

(3) If $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

(4) If $u = \frac{x+y}{1-xy}, v = \tan^{-1}x + \tan^{-1}y$ then find $\frac{\partial(u, v)}{\partial(x, y)}$.

(5) If $\overline{f} = x^2 y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$ then find $curl\,\overline{f}$ and $div\,\overline{f}$.

(6) If $\overline{f}$ is solenoidal function then find a where

$$\overline{f} = (ax+3y+4z)\hat{i} + (x-2y+3z)\hat{j} + (3x+2y-z)\hat{k}.$$

(c) Attempt any two : **10**

(1) If $f(x,y)=\frac{1}{\sqrt{y}}e^{\frac{-(x-a)^2}{4y}}$ then show that $f_{xy}=f_{yx}$.

(2) If $f(x,y)=0$ then obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(3) Expand $e^x \sin y$ in power of x and y.

(4) If $\frac{4}{x}+\frac{9}{y}+\frac{16}{z}=25$ then by Lagrange's method, obtain the value of x,y,z which make $x+y+z$ minimum.

(5) Prove that $\nabla^2(\phi\psi)=\phi\nabla^2\psi+2\nabla\phi\psi+\psi\nabla^2\phi$.

3 (a) Attempt any three : **6**

(1) Find : $\int_0^1\int_0^x e^{\frac{y}{x}}\,dx\,dy$.

(2) Find $\int_c xy\,dx+\left(x^2+y^2\right)dy$ where C : Region of $y=x^2-4$ from (2,0) to (4,12).

(3) Prove that the line integral $\int_{(0,0)}^{(x,y)}\left(6xy^2-y^3\right)dx+\left(6x^2y-3xy^2\right)dy$ is independent of path.

(4) Prove that $p\beta(p,q+1)=q\beta(p+1,q)$.

(5) $u=(x_1,x_2),\ v=(y_1,y_2)\in R^2$. Determine whether $u\cdot v=2x_1y_1+5x_2y_2$ is inner product on R^2.

(6) V is inner product space and $u,v\in V$. Prove that $\|u+v\|^2+\|u-v\|^2=2\|u\|^2+2\|v\|^2$.

(b) Attempt any three : **9**

(1) Find area of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ (a>b)$ by double integral.

(2) Find value of $\iint_R \left(1-x^2-y^2\right) dx\,dy$, where R is circle $x^2+y^2 \le 1$.

(3) Verify the Green's theorem for the line integral $\oint_c x^2 y\,dx + xy^2 dy$, where C is the first quadrant bounded by $y=x$ and $y^3=x^2$.

(4) Find : $\int_0^\infty \frac{x^8\left(1-x^6\right)}{(1+x)^{24}}dx$.

(5) State and prove Triangle inequality in inner product space.

(6) If u and v are vectors in a real inner product space, then show that

$\|u+v\|^2=\|u\|^2+\|v\|^2$ iff $u \perp v$.

(c) Attempt any two : **10**

(1) Find $\iint_R e^{x^2+y^2} dx\,dy$, where R is $x^2+y^2 \le 1$.

(2) State and prove Green's theorem for a plane.

(3) Using Stokes theorem find

$\iint_S x^3 dy\,dz + y^3 dz\,dx + z^3 dx\,dy$, where S is a surface of sphere $x^2+y^2+z^2=a^2$.

(4) State and prove relation between Beta and Gamma functions.

(5) Use Gram-Schmidt process to orthonormalize basis set $\{(1,0,2),(2,0,-1),(0,3,4)\}$ of R^3 with usual (standard) inner product.